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*The unstacking duel and the  
polynomial challenge*

*Science Academies Education Programme  
Lecture Workshop, St. Joseph's College for Women (A)*

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Indian Statistical Institute Bangalore and IIT Gandhinagar

Vishakhapatnam (Feb 27 and 28, 2026)





## *A Curious Claim*

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All horses have the same color.

We will “prove” this using mathematical induction.

Claim: In any set of  $n$  horses, all horses have the same color.

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$n = 1$

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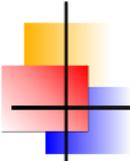
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## Inductive Step

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Assume the statement holds for  $n$  horses, that is,

Any set of  $n$  horses has all horses of the same color

Now consider a set of  $n + 1$  horses:

$$\{H_1, H_2, \dots, H_{n+1}\}.$$

Remove the last horse:  $\{H_1, H_2, \dots, H_n\}$

By the induction hypothesis, these  $n$  horses all have the same color.

Now remove the first horse:  $\{H_2, H_3, \dots, H_{n+1}\}$

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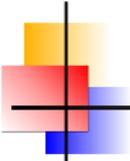
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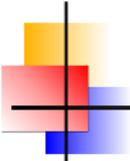
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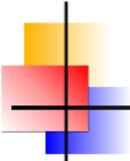
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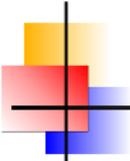
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The two sets overlap in the horses

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So, the colors must match. Therefore,

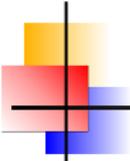
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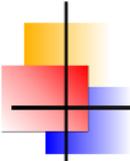
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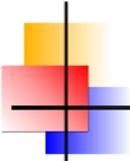
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The problem occurs when we try to prove the step

$$n = 1 \rightarrow n = 2$$

For  $n = 1$  the argument becomes: First set  $\{H_1\}$  and the second set  $\{H_2\}$ . There is **no overlap**.

So we cannot conclude that  $H_1$  and  $H_2$  have the same color!

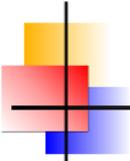
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## *The unstacking game*

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Given  $(n + 1)$  bricks stacked one on top of the other, split them into a set of two piles, one of them consisting of  $n_1$  bricks and the other  $n_2$  bricks such that  $n_1 + n_2 = n + 1$ .

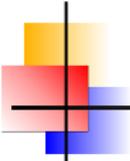
Suppose  $n = 6$ , then this first step may look like this:

You get  $5 \times 2 = 10$  points for this first step

Now, you continue the game by splitting one of the two piles again, say the one with 5 bricks into two piles of 4 bricks and 1 brick:

This time, you earn  $4 \times 1 = 4$  points.





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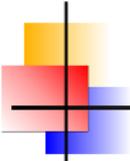
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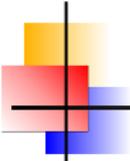
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## *ending the game*

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The game ends when each pile has only one brick left.

The number of points you earn is the total of the points you have earned at each step.

Goal: Given a  $n$ , find a strategy to maximize the number of points earned.

Finishing the game with six bricks, you might end up with

$$5 \times 2 + 4 \times 1 + 2 \times 2 + 1 \times 1 + 1 \times 1 + 1 \times 1 = 21.$$





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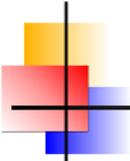
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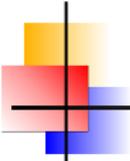
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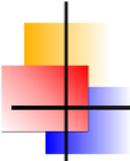
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## Question

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Big Question: Can you do better? Can you do any worse? A different way you could play the game with any  $n$  is the following.

$n + 1 \rightarrow 1, n,$       Points earned:  $n$

$n \rightarrow 1, n - 1,$       Points earned:  $n - 1$

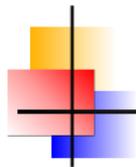
$n - 1 \rightarrow 1, n - 2,$       Points earned:  $n - 2$

$\vdots$                        $\vdots$

Continuing in this manner, after  $n$  steps, we will have nothing to split. The points we would have earned in the bargain is

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}.$$





*let's play*

---

## The unstacking duel



### *Theorem*

*No matter what strategy is used, the score for the unstacking game with  $n + 1$  blocks is  $\frac{n(n+1)}{2}$ .*

The proof is by the principle of strong induction:

Let  $P(n)$  be a property that applies to natural numbers.

Suppose that the following are true:  $P(0)$  is true. For any  $k$ , if  $k \in \mathbb{N}$ ,  $P(0), P(1), \dots, P(k)$  are true, then  $P(k + 1)$  is true.

Then for any  $n \in \mathbb{N}$ ,  $P(n)$  is true.



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For our base case, we prove  $P(0)$ , that any strategy for the unstacking game with one block will always yield  $\frac{0(0+1)}{2} = 0$  points.

This is true because the game immediately ends if the only stack has size one, so all strategies immediately yield 0 points.

For the inductive hypothesis, assume that for some  $n \in \mathbb{N}$  and all  $k \in \mathbb{N}, k \leq n, P(k)$  holds.

Under this hypothesis, to show that  $P(n+1)$  holds.



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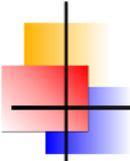
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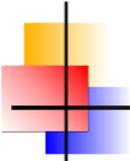
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Since each stack must have at least one block in it, this means that  $k \geq 0$  (so that  $k + 1 \geq 1$ ) and that  $k \leq n$  (so that  $(n - k) + 1 \geq 1$ ).

Consequently, we know that  $0 \leq k \leq n$ , and by the inductive hypothesis we have that the total number of points earned from splitting the stack of  $(k + 1)$  blocks down must be  $\frac{k(k+1)}{2}$ .

Similarly, since  $0 \leq n - k \leq n$ , again by the inductive hypothesis, the total score for the stack of  $(n - k) + 1$  blocks must be  $\frac{(n-k)(n-k+1)}{2}$ .





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The two subgames yield  $\frac{k(k+1)}{2}$  and  $\frac{(n-k+1)(n-k)}{2}$  points, respectively. This means that the total number of points earned is

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# Playing with polynomials!





## The Polynomial

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Consider the polynomial

$$p(x) = a_0 + a_1x + a_2x^2 + \cdots + a_nx^n.$$

The coefficients satisfy

$$0 \leq a_i \leq 2.$$

All coefficients are integers between 0 and 2.

What if all coefficients are integers between 0 and  $k$ ,  
 $k \in \mathbb{N}$ ?





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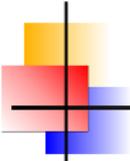
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$$0 \leq a_i \leq 2.$$

All coefficients are integers between 0 and 2.

What if all coefficients are integers between 0 and  $k$ ,  
 $k \in \mathbb{N}$ ?





# The Polynomial

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Consider the polynomial

$$p(x) = a_0 + a_1x + a_2x^2 + \cdots + a_nx^n.$$

The coefficients satisfy

$$0 \leq a_i \leq 2.$$

All coefficients are integers between 0 and 2.

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## The Challenge

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Pick a polynomial  $p$  such that

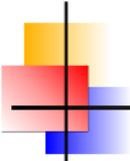
$$p(x) = a_0 + a_1x + a_2x^2 + \cdots + a_nx^n$$

The Challenge: Determine all the coefficients.

You are allowed to ask only one question.

For instance, you may ask for the value of the polynomial at some  $k \in \mathbb{N}$ .





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*let's play again*

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Ask for  $p(7)$ , or perhaps  $p(3)$ , what is the difference?

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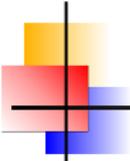
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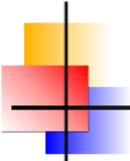
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Which value of  $x$  would reveal the coefficients?

Try  $x = 3$





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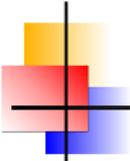
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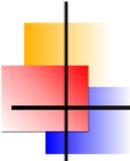
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## Evaluate at 3

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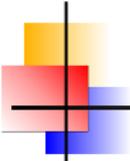
$$p(3) = a_0 + a_1 \cdot 3 + a_2 \cdot 3^2 + \cdots + a_n \cdot 3^n$$

Let

$$p(3) = D$$

Now write  $D$  in base 3.





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## Extracting the Coefficients

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Suppose the answer to our question is  $p(3) = D = 1420$

Write this number in base 3:

$$1420_{10} = 1 \cdot 3^6 + 2 \cdot 3^5 + 2 \cdot 3^4 + 1 \cdot 3^3 + 1 \cdot 3^2 + 2 \cdot 3 + 1$$

In other words,

$$1420_{10} = (1221121)_3.$$

But

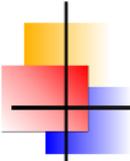
$$p(3) = a_0 + a_1 3 + a_2 3^2 + \cdots + a_n 3^n.$$

Therefore,

$$(a_0, a_1, a_2, a_3, a_4, a_5, a_6) = (1, 2, 1, 1, 2, 2, 1)$$

The base-3 digits are exactly the coefficients.





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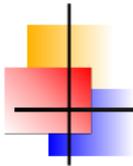
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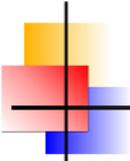
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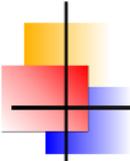
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## Why does this work?

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Recall that a number written in base 3 has the form

$$D = d_0 + d_13 + d_23^2 + \cdots + d_n3^n,$$

where

$$0 \leq d_i \leq 2.$$

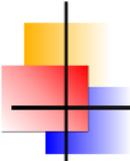
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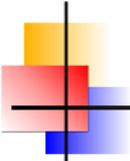
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## *Conclusion*

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Ask for the value

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Write  $D$  in base 3.

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Only one question is needed.





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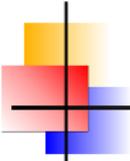
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Suppose instead

$$0 \leq a_i \leq k - 1$$

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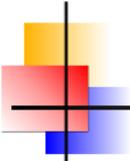
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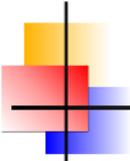
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## Two Questions

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Suppose that the coefficients are arbitrary natural numbers  $a_i \in \mathbb{N}$ .

It is not enough to ask just one question to determine the polynomial. Is it enough to ask two?

What if someone gives you only the value of  $p$  at  $\pi$  without giving you a choice?





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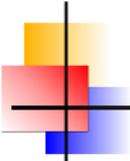
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Thank You!

