The unstacking duel and the polynomial challenge

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Gadadhar Misra

Indian Statistical Institute Bangalore and IIT Gandhinagar

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A Curious Claim

All horses have the same color.

We will “prove” this using mathematical induction.

Claim: In any set of n horses, all horses have the same color.

The base case: Consider a set containing only one horse.

$n = 1$

In a set with a single horse, all horses clearly have the same color. So the statement holds for $n = 1$.

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Inductive Step

Assume the statement holds for n horses, that is,

Any set of n horses has all horses of the same color

Now consider a set of $n+1$ horses:

$$\{H_1, H_2, \ldots, H_{n+1}\}.$$

Remove the last horse: $\{H_1, H_2, \ldots, H_n\}$

By the induction hypothesis, these n horses all have the same color.

Now remove the first horse: $\{H_2, H_3, \ldots, H_{n+1}\}$

Again, these n horses all have the same color.

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The Conclusion

The two sets overlap in the horses

$$H_2, \ldots, H_n.$$

So, the colors must match. Therefore,

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By induction, all horses have the same color.

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Something Is Wrong

The problem occurs when we try to prove the step

$$n = 1 \rightarrow n = 2$$

For $n = 1$ the argument becomes: First set $\{H_1\}$ and the second set $\{H_2\}$. There is no overlap.

So we cannot conclude that H_1 and H_2 have the same color!

The inductive step silently assumed that the two sets

$$\{H_1, \ldots, H_n\} \quad \text{and} \quad \{H_2, \ldots, H_{n+1}\}$$

share at least one horse. This fails when $n = 1$.

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The unstacking game

Given $(n+1)$ bricks stacked one on top of the other, split them into a set of two piles, one of them consisting of n_1 bricks and the other n_2 bricks such that $n_1 + n_2 = n + 1$.

Suppose $n = 6$, then this first step may look like this:

You get $5 \times 2 = 10$ points for this first step

Now, you continue the game by splitting one of the two piles again, say the one with 5 bricks into two piles of 4 bricks and 1 brick:

This time, you earn $4 \times 1 = 4$ points.

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ending the game

The game ends when each pile has only one brick left.

The number of points you earn is the total of the points you have earned at each step.

Goal: Given a n, find a strategy to maximize the number of points earned.

Finishing the game with six bricks, you might end up with

$$5 \times 2 + 4 \times 1 + 2 \times 2 + 1 \times 1 + 1 \times 1 + 1 \times 1 = 21.$$

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Question

Big Question: Can you do better? Can you do any worse? A different way you could play the game with any n is the following.

$$\begin{aligned} n+1 &\to 1, n, && \text{Points earned: } n \\ n &\to 1, n-1, && \text{Points earned: } n-1 \\ n-1 &\to 1, n-2, && \text{Points earned: } n-2 \\ &\vdots && \vdots \end{aligned}$$

Continuing in this manner, after n steps, we will have nothing to split. The points we would have earned in the bargain is

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}.$$

let's play

The unstacking duel

Theorem

No matter what strategy is used, the score for the unstacking game with $n+1$ blocks is $\frac{n(n+1)}{2}$.

The proof is by the principle of strong induction:

Let $P(n)$ be a property that applies to natural numbers. Suppose that the following are true: $P(0)$ is true. For any k, if $k \in \mathbb{N}, P(0), P(1), \ldots, P(k)$ are true, then $P(k+1)$ is true.

Then for any $n \in \mathbb{N}, P(n)$ is true.

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the proof

For our base case, we prove $P(0)$, that any strategy for the unstacking game with one block will always yield $\frac{0(0+1)}{2} = 0$ points.

This is true because the game immediately ends if the only stack has size one, so all strategies immediately yield 0 points.

For the inductive hypothesis, assume that for some $n \in \mathbb{N}$ and all $k \in \mathbb{N}, k \leq n, P(k)$ holds.

Under this hypothesis, to show that $\mathrm{P(n+1)}$ holds.

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finishing the proof

Since each stack must have at least one block in it, this means that $k \geq 0$ (so that $k+1 \geq 1$) and that $k \leq n$ (so that $(n-k)+1 \geq 1$).

Consequently, we know that $0 \leq k \leq n$, and by the inductive hypothesis we have that the total number of points earned from splitting the stack of ($k+1$) blocks down must be $\frac{k(k+1)}{2}$.

Similarly, since $0 \leq n-k \leq n$, again by the inductive hypothesis, the total score for the stack of $(n-k)+1$ blocks must be $\frac{(n-k)(n-k+1)}{2}$.

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Let us consider the total score for this game. The initial move yields $(k+1)(n-k+1)$ points.

The two subgames yield $\frac{k(k+1)}{2}$ and $\frac{(n-k+1)(n-k)}{2}$ points, respectively. This means that the total number of points earned is

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This is the answer with a stack of size $n+2$. The inductive step is therefore verified, completing the proof.

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Playing with polynomials!

The Polynomial

Consider the polynomial

$$p(x) = a_0 + a_1x + a_2x^2 + \cdots + a_nx^n.$$

The coefficients satisfy

$$0 \leq a_i \leq 2.$$

All coefficients are integers between 0 and 2.

What if all coefficients are integers between 0 and k, $k \in \mathbb{N}$?

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The Challenge

Pick a polynomial p such that

$$p(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n$$

The Challenge: Determine all the coefficients.

You are allowed to ask only one question.

For instance, you may ask for the value of the polynomial at some $k \in \mathbb{N}$.

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The polynomial challenge

Ask for $p(7)$, or perhaps $p(3)$, what is the difference?

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What should we ask?

If we evaluate the polynomial at some number x we obtain

$$p(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n$$

Which value of x would reveal the coefficients?

Try $x = 3$

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Evaluate at 3

Suppose that

$$p(3) = a_0 + a_1 \cdot 3 + a_2 \cdot 3^2 + \cdots + a_n \cdot 3^n$$

Let

$$p(3) = D$$

Now write D in base 3.

Evaluate at 3

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Extracting the Coefficients

Suppose the answer to our question is $p(3) = D = 1420$

Write this number in base 3:

$$1420_{10} = 1 \cdot 3^6 + 2 \cdot 3^5 + 2 \cdot 3^4 + 1 \cdot 3^3 + 1 \cdot 3^2 + 2 \cdot 3 + 1$$

In other words,

$$1420_{10} = (1\,2\,2\,1\,1\,2\,1)_3.$$

But

$$p(3) = a_0 + a_1 3 + a_2 3^2 + \cdots + a_n 3^n.$$

Therefore,

$$(a_0, a_1, a_2, a_3, a_4, a_5, a_6) = (1, 2, 1, 1, 2, 2, 1)$$

The base-3 digits are exactly the coefficients.

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The base-3 digits are exactly the coefficients.

Why does this work?

Recall that a number written in base 3 has the form

$$D = d_0 + d_1 3 + d_2 3^2 + \cdots + d_n 3^n,$$

where

$$0 \le d_i \le 2.$$

Compare this with

$$p(3) = a_0 + a_1 3 + a_2 3^2 + \cdots + a_n 3^n.$$

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Conclusion

Ask for the value

$$D = p(3).$$

Write D in base 3.

The base-3 digits give

$$a_0, a_1, \ldots, a_n$$

Only one question is needed.

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A Natural Generalization

Suppose instead

$$0 \leq a_i \leq k - 1$$

Then ask for

$$p(k)$$

Write $p(k)$ in base k.

The digits give the coefficients.

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The digits give the coefficients.

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It is not enough to ask just one question to determine the polynomial. Is it enough to ask two?

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Thank You!